FLOW INTO A VACUUM ALONG AN OBLIQUE WALL

(ISTECHENIE V VAKUUM NA KOSOI STENKE)

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A polytropic gas with $1 < \gamma < 3$ fills the domain $x_1 > 0$, $x_2 - x_1 \tan \alpha > 0$, which is bounded by rigid walls. At $t = 0$ the wall $x_1 = 0$ is removed, and the gas begins to flow into the vacuum.

The motion is a self-similar potential flow. It may be assumed that the velocity components u_1 and u_2, the speed of sound c, and the similarity variables $\xi_1 = x_1/t$, $\xi_2 = x_2/t$ are referred to the speed of sound in the gas at rest, $c_0 = 1$.

Far away from the wall $\xi_2 = \xi_1 \tan \alpha$, the motion is just an undisturbed Riemann wave

$$u_1 = \frac{\xi_1 - 1}{\varkappa + 1}, \qquad u_2 = 0, \qquad c = 1 + \varkappa u_1, \qquad \varkappa = \frac{\gamma - 1}{2}, \qquad -\frac{1}{\varkappa} \leqslant \xi_1 \leqslant 1$$

In the domain where the disturbance occurs, let us seek a solution in the form of a double wave [1]. The problem reduces to the determination of a function $c(u_1, u_2)$ which satisfies the equation

$$(\varkappa^2 - c_2^2)\, c_{11} + 2c_1c_2c_{12} + (\varkappa^2 - c_1^2)\, c_{22} = (\varkappa / c)\, [(c_1^2 + c_2^2)(1 - \varkappa) - 2\varkappa^2] \qquad (1)$$

together with the boundary conditions

$$u_2 = 0, \ c = 1 + \varkappa u_1, \qquad -\varkappa^{-1} \leqslant u_1 \leqslant 0, \qquad u_2 = u_1 \tan\alpha, \qquad c_2 = c_1 \tan\alpha \qquad (2)$$

where

$$c_i = \frac{\partial c}{\partial u_i}, \qquad c_{ij} = \frac{\partial^2 c}{\partial u_i \partial u_j} \qquad (i, j, = 1, 2)$$

The line $u_2 = 0$ is a characteristic for equation (1). From the equation satisfied by characteristics, we find for $0 < \kappa < 1$, that

$$c_2 = \varkappa\,[\beta + (1 + \varkappa u_1)^{(1-\varkappa)/\varkappa}(\tan^2\alpha - \beta)]^{1/2} \qquad \left(\beta = \frac{1+\varkappa}{1-\varkappa} > 1\right)$$

The second characteristic passing through the point $u_1 = u_2 = 0$ lies outside the domain of integration, that is, we have here the second mixed boundary value problem for equation (1), whose solution may be obtained by the method of characteristics [2]. The domain of integration is bounded on the left by the characteristics passing through the point $u_1 = -\kappa^{-1}$, $u_2 = 0$.

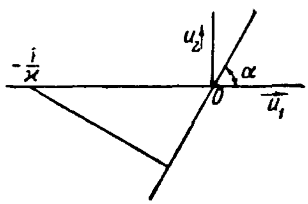

Fig. 1.

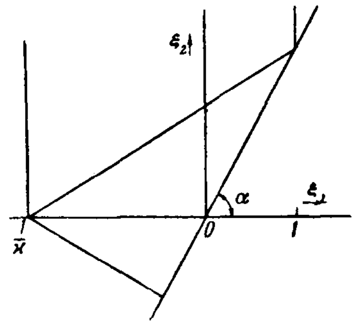

Fig. 2.

The boundary curve of the vacuum is given by the equation $c(u_1, u_2) = 0$. The slopes of the tangents to the boundary characteristic and to the curve $c(u_1, u_2) = 0$ at the point $u_1 = -\kappa^{-1}$, $u_2 = 0$, are $-2\sqrt{\beta}/(\beta - 1)$ and $-1/\sqrt{\beta}$ respectively. Therefore, in the neighborhood of this point the curve $c(u_1, u_2) = 0$ must lie inside the domain of integration. Clearly, the curve $c(u_1, u_2) = 0$ intersects the wall at a right angle. Once the function $c(u_1, u_2)$ has been obtained as a solution of the boundary value problem (1) and (2), the passage from the hodograph plane (the u_1u_2 plane) to the physical plane (the $\xi_1\xi_2$ plane) is carried out by means of the formulas

$$\xi_i = u_i + \frac{cc_i}{\varkappa} \qquad (i = 1, 2)$$

When the angle α is given, in terms of the polytropic exponent γ by means of the equation $\beta = \tan^2\alpha\,(\beta > 1,\ \alpha > 1/4\,\pi)$, then the solution of the boundary value problem (1) and (2) is

$$c = 1 + \varkappa u_1 + \varkappa \sqrt{\beta}\, u_2$$

The boundary of the disturbed domain in common with the domain of the vacuum, $u_1 + \sqrt{\beta} u_2 = -\kappa^{-1}$ is therefore inside the domain of integration.

The solution in the physical plane then has the form

$$c = \beta^{-1}(1 + \varkappa\xi_1 + \varkappa\sqrt{\beta}\xi_2), \qquad u_1 = \xi_1 - c, \qquad u_2 = \xi_2 - \sqrt{\beta}c$$

For the boundary common to the domain of the disturbance and the domain of the Riemann wave and the vacuum respectively, we obtain

$$\xi_1 - \sqrt{1-\varkappa^2}\xi_2 = -\varkappa^{-1}, \qquad \xi_1 + \sqrt{\beta}\xi_2 = -\varkappa^{-1}$$

Figures 1 and 2 represent the flow in the u_1u_2 and the $\xi_1\xi_2$ planes, in the special case $\alpha = 1/3\ \pi$, $\gamma = 2$.

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